JAIN COLLEGE V V Puram **2nd PUC MOCK Paper – Jan. 2024**	**Course:** 2nd year PUC **Subject:** Mathematics **Max. Marks:** 80 **Duration:** 3:15 hour

Instructions : 1. The question paper has five parts namely A, B, C, D and E
Answer all the parts
2. Part A has 15 multiple choice questions, 5 fill in the blank questions
3. Use the graph sheet for question on linear programming problem in part E

PART A

I. Answer all the multiple-choice questions: **15 × 1 = 15**

1. The relation R in the set {1,2,3} given by R={(1,2), (2,1), (1,1)} is

   a) Reflexive and transitive   b) Symmetric and transitive   c) Only symmetric   d) Equivalence

2. The signum function $f: R \to R$ given by $f(x) = \begin{cases} 1 & if\ x > 0 \\ 0 & if\ x = 0 \\ -1 & if\ x < 0 \end{cases}$ is

   a) f is one-one onto   b) f is one-one but not onto

   c) f is not one-one but onto   d) Neither one-one nor onto

3. The principal value branch of $\cos^{-1} x$

   a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ c) $(0, \pi)$ d) $[0, \pi]$

4. The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is

   a) 27   b) 25   c) 81   d) 512

5. If A is an invertible matrix of order 2 and $|A| = 10$ then $\det(A^{-1})$ is equal to

   a) 10   b) $\frac{1}{10}$ c) -10 d) 1

6. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function, is continuous at

   a) 4   b) 1.5   c) -2 d) 1

7. The derivative of $y = \log_{10} x$ is

   a) $\frac{1}{x}$ b) $\frac{1}{x(\log_e 10)}$ c) $\frac{\log x}{\log 10}$ d) $\log_x 10$

8. The interval in which $y = x^2 e^{-x}$ is increasing is

   a) $(-\infty, \infty)$ b) $(-2, 0)$ c) $(2, \infty)$ d) $(0, 2)$

9. $\int \sin(3x-2)dx$ is

   a) $-\frac{\sin(3x-2)}{2} + c$ b) $\frac{\cos(3x-2)}{2} + c$ c) $-\frac{\cos(3x-2)}{3} + c$ d) $-\frac{\sin(3x-2)}{3} + c$

10. $\int_0^2 e^x dx$ is

    a) e^2 b) e^x c) $e^2 - 1$ d) $e^x - 2$

11. If $\vec{a}$ is a non-zero vector of magnitude ‘a’ and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector, if

    a) $\lambda = 1$ b) $\lambda = -1$ c) $a = |\lambda|$ d) $a = \frac{1}{|\lambda|}$

12. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is Equal to

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π

13. If a line makes angles α, β, γ with the positive direction co-ordinate axes, then the value of $sin^2\alpha + sin^2\beta + sin^2\gamma$ is

a) b) 1 c) 4 d) 0

14. The minimum value of the objective function $z = 4x + 3y$ occurs at the point

a) (0, 8) b) (2,5) c) (4,3) d) (9,0)

15. The probability of obtaining an even prime number on each die , when a pair of dice are rolled is

a) 0 b) $\frac{1}{3}$ c) $\frac{1}{12}$ d) $\frac{1}{36}$

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket

5 × 1 = 5

$\left(0, \ 2, \ 1, \ \frac{\pi}{4}, \ 7, \ \frac{\pi}{2}\right)$

16. The value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ is ________________

17. A square matrix A is a singular matrix if |A| is ___________

18. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ is __________

19. The lines $\frac{x-5}{k} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular, then k is_______

20. If E and F be events of a sample space S of an experiment then $P(S\,|\,F)$ is _____

PART –B

III. Answer any six questions **6 × 2 =12**

21. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{7}$

22. Find the equation of line joining (1, 2) , (3, 6) using determinant method

23. Find $\frac{dy}{dx}$, if $2x + 3y = \sin y$.

24. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

25. Find the local minimum value of the function f given by $f(x) = |x + 2| - 1$

26. Find $\int \frac{1}{(x+1)(x-2)}dx$

27. Evaluate $\int_0^{\frac{\pi}{2}} \cos 2x\,dx$

28. Find the angle between the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

29. Find the angle between the pair of lines given by

$$\vec{r}=\left(3\hat{i}+2j-4k\right)+\lambda\left(\hat{i}+2j+2k\right) \text{ and } \vec{r}=\left(5\hat{i}-2j\right)+\mu\left(3\hat{i}+2j+6k\right)$$

30. Two coins are tossed once. Find $P(E|F)$ where E : no tail appears, F : no head appears.

31. If A and B are two independent events , then prove that the probability of occurrence of at least one of A and B is given by $1-P(A')\,P(B')$.

PART – C

IV. Answer any six questions **6 × 3 =18**

32. Show that the relation R in the set of real numbers R defined as R= $\{(a,b): a \le b^2\}$ is neither reflexive nor symmetric nor transitive.

33. Simplify: $\tan^{-1}\left(\frac{2\cos x-3\sin x}{3\cos x+2\sin x}\right)$, $\frac{2}{3}\tan x>-1$.

34. If A and B are invertible matrices of the same order, then prove that $(AB)^{-1}=B^{-1}A^{-1}$.

35. If $x=\sqrt{a^{\sin^{-1}t}}$, $y=\sqrt{a^{\cos^{-1}t}}$ show that $\frac{dy}{dx}=-\frac{y}{x}$.

36. Differentiate $x^{\log x}$, x > 0 with respect to x

37. Find the intervals in which the function $f(x) = x^2 - 4x + 6$ is

i) strictly increasing ii) strictly decreasing

38. Find $\int x(\log x)^2 dx$

39. Find the equation of curve passing through the point (–2, 3) , given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$

40. Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $|\vec{a}|=1$, $|\vec{b}|=4$, $|\vec{c}|=2$

41. If $\vec{a}=2\hat{i}+2j+3k$, $\vec{b}=-\hat{i}+2j+k$ and $\vec{c}=3\hat{i}+j$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ

42. A man is known to speak truth 4 out of 5 times. He tossed a coin and reports that it is head. Find the probability that it is actually head?

PART – D

V. Answer any four questions **4 × 5 = 20**

43. Let f : N→ Y be a function defined as f(x) = 4x + 3, where Y = {y ∈ N : y = 4x + 3 for some x ∈ N}.Show that f is invertible. Find the inverse of f.

44. If A = $\begin{bmatrix}1\\3\\-2\end{bmatrix}$ B = $\begin{bmatrix}2 & 4 & -6\end{bmatrix}$ verify that $(AB)' = B'A'$

45. Solve by matrix method: $x+y+z=6$, $x-2y+3z=6$, $x-y+z=2$.

46. If $y=\sin^{-1}x$, $(-1\le x\le 1)$ Show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$.

47. Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to x and hence evaluate $\int\frac{dx}{\sqrt{x^2+2x+4}}$.

48. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

49. Solve the differential equation $\cos^2 x\frac{dy}{dx}+y=\tan x$

50. Derive the formula to find the distance between two skew lines.

PART – E

VI. Answer the following questions

51. Prove that $\int_0^a f(x)dx=\int_0^a f(\mathrm{a}-x)dx$ **6 marks**

And hence evaluate $\int_0^{\pi/4}\log(1+\tan x)\ dx$

OR

Maximize and Minimize $Z = 3x + 2y$ subject to the constraints

$x + 2y \le 10,\ \ 3x + y \le 15\ \ , x \ge 0,\ \ y \ge 0$

52. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ Show that $A^2-5A+7I=O$. Hence find A^{-1} **4 marks**

OR

Find the value of k so that the function $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x} & if\ \ x\neq\frac{\pi}{2}\\ 3 & if\ x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$

***